

Exercise 37

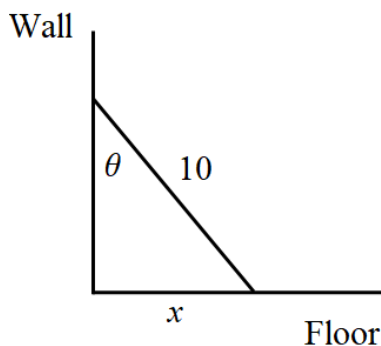
A ladder 10 ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \pi/3$?

Solution

The aim of this problem is to determine how fast x changes with respect to θ when $\theta = \pi/3$, that is,

$$\left. \frac{dx}{d\theta} \right|_{\theta=\pi/3}.$$

What we need is a formula for x in terms of θ . Start by drawing a picture of the ladder against the wall.



$$\sin \theta = \frac{x}{10}$$

Solve for x .

$$x = 10 \sin \theta$$

Take the derivative with respect to θ .

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(10 \sin \theta) = 10 \cos \theta$$

Evaluate it at $\theta = \pi/3$.

$$\left. \frac{dx}{d\theta} \right|_{\theta=\pi/3} = 10 \cos \frac{\pi}{3} = 5 \frac{\text{ft}}{\text{rad}}$$